The Design of High-Order Bandpass Sigma-Delta Modulators Using Low-Spread Single-Stage Structure

Hwi-Ming Wang and Tai-Haur Kuo

Abstract—A structure for single-stage high-order bandpass sigma-delta modulators (BPSDMs) is presented. The proposed structure introduces an additional internal path in each resonator, thus, adding one degree of freedom in coefficient determination. Coefficient spread can therefore be reduced, resulting in significantly reduced capacitance area in switched-capacitor BPSDM circuits. High-order BPSDMs with different quality factors \((Q)\) are demonstrated. It shows that coefficient spread is significantly reduced, especially for high-\(Q\) applications. For comparable eighth-order 3-bit BPSDMs, the maximum coefficient spread are respectively 15369 and 7693 for conventional cascade-of-resonator-with-feedback (CRFB) and cascade-of-resonator-with-feedback-forward (CRFF) designs, and 114 for the proposed structure. For an eighth-order 1-bit example, these respective values are 8994, 2638, and 74. With coefficient mismatch, peak signal-to-noise ratio (PSNR) degradation of the proposed structure is less than those of the CRFB and CRFF structures, demonstrating reduced sensitivity to component mismatch. Hence, the proposed structure can reduce chip area and ease circuit implementation of BPSDMs.

Index Terms—Bandpass Sigma–Delta modulator, cascade-of-resonator structure, low-coefficient spread.

I. INTRODUCTION

LOW-PASS Sigma-Delta modulators (SDMs) have revolutionized the design and implementation of high-resolution baseband data converters [1]. Bandpass sigma-delta modulators (BPSDMs) [2] extend the concept of the lowpass SDMs to intermediate-frequency (IF) signals. To digitize signals at intermediate frequency, BPSDMs offer potential benefits including reduced sensitivity to analog circuit imperfections, fewer off-chip components and the advantages due to CMOS scaling.

BPSDMs with high-dynamic range (DR) generally use high-order loop filter, multibit quantizer and/or high-oversampling ratio (OSR), where OSR is defined as the ratio of sampling frequency to two times of signal bandwidth. To construct high-order BPSDMs, both multistage [3] and single-stage [2] structures can be used. The multistage structure usually yields more efficient noise shaping characteristics, but component mismatch degrades its performance. The single-stage structure is less sensitive to component mismatch and is adopted in this paper. A BPSDM with high OSR increases not only circuit speed but also the quality factor \((Q)\) of BPSDM resonators. In switched-capacitor (SC) circuits, the capacitor spread, which is the ratio between the largest and the smallest capacitance, is proportional to \(Q\) [4]. In addition, larger capacitor spread results in larger capacitance area for circuit realization. Hence, total capacitance becomes extravagant when \(Q\) becomes high. In this paper, a new single-stage structure for BPSDMs is proposed. Compared to other single-stage BPSDM structures, the proposed structure has lower coefficient spread. Further, the proposed structure also reduces sensitivity to process variation. Hence, the proposed structure can reduce chip area and ease circuit implementation of BPSDMs.

This paper is organized as follows. Section II shows the coefficient spread in conventional single-stage structures, and a new low-spread structure is presented. In Section III, performance of BPSDMs using conventional and the proposed structures are compared, including coefficient spread, capacitor spread and performance degradation due to process variation. Conclusions are presented in Section IV.

II. BPSDM STRUCTURES

A BPSDM consists of a quantizer, a bandpass loop filter and a feedback digital-to-analog converter (DAC). The transfer function of a BPSDM can be derived as

\[ Y(z) = STF(z)X(z) + NTF(z)N(z) \]

where \(X(z)\) is input signal, \(N(z)\) is noise, \(Y(z)\) is output signal, \(STF(z)\) is signal transfer function and \(NTF(z)\) is noise transfer function. \(NTF(z)\) can be approximated by conventional Inverse–Chebyshev bandreject analog filter functions. After considering the stability, coefficient tolerance for circuit component mismatches and design tradeoffs among in-band quantization noise, OSR, modulator order and quantizer bit number, an appropriate \(NTF(z)\) can be obtained [5].

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A. NTF(z) Coefficient Spread of High-Q BPSDMs

For a BPSDM, the NTF(z) of the BPSDM with a modulator order of \( N \) can be decomposed as the multiplication of \( N/2 \) biquadratic equations

\[
\text{NTF}(z) = \prod_{i=1}^{N/2} \left( \frac{z^2 - \frac{4Q_{b1} \cos(\omega_{b1}T)}{2Q_{b1} + \sin(\omega_{b1}T)} z + \frac{2Q_{b1} \sin(\omega_{b1}T)}{2Q_{b1} + \sin(\omega_{b1}T)}}{z^2 - \frac{4Q_{b1} \cos(\omega_{b1}T)}{2Q_{b1} + \sin(\omega_{b1}T)} z + \frac{2Q_{b1} \sin(\omega_{b1}T)}{2Q_{b1} + \sin(\omega_{b1}T)}} \right)
\]

where both \( \omega_{b1} \) and \( \omega_{b1} \) are the resonant frequencies, and both \( Q_{b1} \) and \( Q_{b1} \) are quality factors of numerator and denominator in the \( i \)th biquadratic equation, respectively. For high-Q BPSDMs with the ratio of sampling frequency to central frequency equal to 4, \( Q_{b1} \) and \( Q_{b1} \) are high and \( \omega_{b1} \) and \( \omega_{b1} \) are close to the central frequency \( (\pi/2) \). This makes the odd-order coefficients of the biquadratic equation far less than 1, i.e.

\[
\frac{4Q_{b1} \cos(\omega_{b1}T)}{2Q_{b1} + \sin(\omega_{b1}T)} \ll 1 \quad \text{and} \quad \frac{4Q_{b1} \cos(\omega_{b1}T)}{2Q_{b1} + \sin(\omega_{b1}T)} \ll 1.
\]

Hence, the odd-order coefficients of the NTF(z) numerator and denominator are smaller than the even-order coefficients. This will cause large coefficient spread.

B. Conventional BPSDM Structures

Many different single-stage structures are available for BPSDM design. These include cascade-of-resonator in feed-
Table I

Upper part: Specifications of design examples I, II, and III; lower part: synthesized BPSDM coefficients and performance of examples I, II, and III.

<table>
<thead>
<tr>
<th>Example</th>
<th>Specifications of design examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Quantizer bits</td>
<td>3</td>
</tr>
<tr>
<td>STF(z) design</td>
<td>programmable</td>
</tr>
<tr>
<td>Minimum PSNR</td>
<td>85dB</td>
</tr>
<tr>
<td>Modulator Orders</td>
<td>8</td>
</tr>
<tr>
<td>Maximum input</td>
<td>-10dB</td>
</tr>
<tr>
<td>Q</td>
<td>15</td>
</tr>
<tr>
<td>Center frequency</td>
<td>15MHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>60MHz</td>
</tr>
</tbody>
</table>

An $N$th-order CRFB structure is shown in Fig. 1(a). An eighth-order one-bit CRFB BPSDM was applied in [2], showing that the odd-order coefficients of $\alpha_{2n+1}$ ($n = 1 \sim N$) in Fig. 1(a) are extremely small, resulting in a large coefficient spread. For a multibit BPSDM, CRFB needs some multibit feedback DACs, which can be an SC circuit. The unit capacitor value for the DAC circuit is inversely proportional to the quantizer bit number (i.e., the capacitor for a 3-bit DAC is $C/7$, but it is $C$ for a 1-bit DAC). Thus, both larger quantizer bit number and higher $Q$ increase capacitor spread, making CRFB a poor choice for multibit high-$Q$ BPSDMs.

An $N$th-order CRFF BPSDM structure is shown in Fig. 1(b). Its NTF(z) can be expressed as

$$\text{NTF}_{\text{CRFF}}(z) = \frac{L(z)}{I(z) + M(z)}$$ (1)
The terms of \( \text{STF}(z) \) are very compressed as shown in Fig. 1(b) are very compressed. Thus, the proposed structure is designated low spread cascade-of-resonator feedforward (LSCRFF) structure.

For a high-order CRFF BPSDM with high-Q, it shows the odd-order coefficients of \( a_{2i} \) (\( n = 1 \sim N \)) in Fig. 1(b) are very small. Hence, it has large coefficient spread, too.

\[ L(z) = \prod_{i=1}^{N/2} [(z-1)^2 + c_i(z-1) + b_i] \quad (2) \]

\[ M(z) = \sum_{i=1}^{N/2} [a_{2i} + c_i a_{2i-1} + a_{2i-1}(z-1)] \]
\[ \cdot \prod_{j=i+1}^{N/2} [(z-1)^2 + c_j(z-1) + b_j] \quad (3) \]

The NTF(z) of an \( N \)-th order LSCRFF BPSDM can be expressed as

\[ \text{NTF}_{\text{LSCRFF}}(z) = \frac{P(z)}{P(z) + Q(z)} \quad (4) \]

where

\[ P(z) = \prod_{i=1}^{N/2} [(z-1)^2 + (c_i + b_i d_i)(z-1) + b_i] \quad (5) \]

\[ Q(z) = \sum_{i=1}^{N/2} [(a_{2i} d_i + a_{2i-1})(z-1) + (a_{2i-1} c_i + a_{2i})] \]
\[ \cdot \prod_{k=1}^{i-1} [1 + d_k(z-1)] \]
\[ \cdot \prod_{j=i+1}^{N/2} [(z-1)^2 + (c_j + b_j d_j)(z-1) + b_j]. \quad (6) \]

For a given NTF(z) implemented by CRFF and LSCRFF structures, \( L(z) \) and \( M(z) \) in CRFF are the same as \( P(z) \) and \( Q(z) \) in LSCRFF, respectively. The terms of \( c_i \) in \( L(z) \) for determining the odd-order coefficients of the NTF(z) numerator are replaced by \( c_i + b_i d_i \) in \( P(z) \). The terms of \( a_{2i-1} \) in \( M(z) \) for determining the odd-order coefficients of the NTF(z) denominator are replaced by \( a_{2i} d_i + a_{2i-1} \) in \( Q(z) \). The \( P(z) \) and \( Q(z) \) in LSCRFF add one degree of freedom in coefficient determination so that coefficient spread can be relaxed.

C. Proposed Low Spread Structure

The proposed structure as shown in Fig. 1(c), is based on the CRFF but contains an extra feedforward path, \( d_i \), in each resonator [6]. The extra path offers an additional degree of freedom in coefficient determination, allowing greatly reduced coefficient spread. Thus, the proposed structure is designated low spread cascade-of-resonator feedforward (LSCRFF) structure.

Fig. 2. (a) Pole/zero diagram. (b) Frequency response of NTF(z) and STF(z).

Fig. 3. Maximum coefficient spread versus \( Q \) for BPSDMs with: (a) 3-bit and programmable STF(z); (b) 3-bit and STF(z) = 1 - NTF(z); and (c) 1-bit and programmable STF(z), where \( \circ \), \( \ast \), and \( \circ \) denote CRFB, CRFF, and LSCRFF, respectively.
III. PERFORMANCE COMPARISONS BETWEEN CONVENTIONAL AND PROPOSED STRUCTURES

Three design examples, whose specifications are summarized in the upper part of Table I, are used to compare CRFB, CRFF, and LSCRFF in terms of coefficient spread, capacitor value and capacitor spread of SC circuits, and also performance degradation due to process variation. In examples I and III, the STF(z) are programmable and zeros are placed symmetrically, with 3 zeros at dc, 3 zeros at π and one at the center of the unit circle. In example II, ak are equal to fik (i = 1 ~ 8) in Fig. 1(a), and all input feedforward paths are removed except fik in Fig. 1(b) and (c). In this case, STF(z) is dependent on NTF(z) according to the relation STF(z) = 1 − NTF(z). The pole and zero positions of STF(z) and NTF(z) are shown in Fig. 2(a), and the frequency responses are in Fig. 2(b). It shows that the out-of-band signal is greatly attenuated and the in-band gain ripple is smaller in example I.

A. Coefficient Spread

The lower part of Table I shows the coefficients of the CRFB, CRFF and LSCRFF for the specifications in the upper part of Table I. Clearly, LSCRFF offers smaller coefficient spread compared to CRFF and CRFB. To verify the improved coefficient spread for different Q and modulator orders, examples for Q = 8 ~ 80 and modulator orders of 4, 6, and 8 are investigated and compared. Fig. 3 shows the relation between the maximum spread and Q for the three examples, and demonstrates that the proposed LSCRFF significantly lowers coefficient spread, es-
especially in high-$Q$ application. For low-$Q$ cases, LSCRFF still retains the advantage, but the extent of improvement is less than for high-$Q$ situations.

**B. Capacitor Spread**

For SC circuit implementation of BPSDMs, the SC resonator shown in Fig. 4(a) is realized by a stray-insensitive, fully differential Forward–Euler loop. The required capacitor values can be determined from the coefficients in Table I. The minimum sampling capacitor is determined by the KT/C noise which should be smaller than the total noise requirement. If the peak signal-to-KT/C ratio ($\text{PSNR}_{\text{KT/C}}$) is 88 dB, then the sampling capacitor of the first resonator stage is at least 1.5 pF ($C_s = 1.5$ pF). This value will be used for the following three design examples. It still leaves 3-dB margin for the noise contributed by other nonidealities. The latter stages have smaller sampling capacitors due to their allowance of higher noise budgets.
The resonator stages in CRFF and LSCRFF are fed to a summing stage whose SC circuit is shown in Fig. 4(b). From Table I, the coefficient spread of $a_n$ ($n = 1 \sim 8$) in CRFF is much larger than in LSCRFF. The $C_{\text{sum}}$ in LSCRFF can be chosen to be smaller than in CRFF. In example I, $C_{\text{sum}}$ is 4200 fF and 20 fF for CRFF and LSCRFF, respectively. After considering the minimum capacitor and noise budget for the resonator stages, the maximum capacitor spreads in example I are 11527, 1572, and 868 for CRFB, CRFF and LSCRFF, respectively. For example II, these respective values are 11316, 1400, and 284. For example III, these respective values are 9445, 2248, and 980. Hence, it shows that LSCRFF has the smallest capacitor spread and the capacitance area.

C. Process Variation

Monte-Carlo simulations with 100 different samples each for CRFB, CRFF, and LSCRFF are used to investigate PSNR degradation due to process variation. Histograms of PSNR deviation with 0.5%, 1%, and 2% maximum mismatch are shown in Fig. 5(a). For the case of 1% maximum mismatch, maximum PSNR degradation is less than 5 dB for LSCRFF, but it is 9 and 11 dB for CRFB and CRFF structures, respectively. The results show that the proposed LSCRFF is less sensitive to capacitor mismatch than CRFB and CRFF. For maximum component variations of 0.5% to 2%, modulator orders of 6 and 8, and quantizer bit numbers of 1 and 3, maximum PSNR degradation is plotted in Fig. 5(b), wherein LSCRFF again shows the least sensitive to capacitor mismatch.

IV. Conclusion

A low-spread structure for high-order BPSDMs is presented. The proposed LSCRFF structure adds one degree of freedom in coefficient determination, thereby lowering coefficient spread compared to conventional CRFB and CRFF structures. For high-$Q$ applications, reduction of coefficient spread becomes significant. In addition, LSCRFF reduces total capacitance area of SC-implemented BPSDMs, and is less sensitive to process variation. Hence, the proposed LSCRFF can reduce chip area and ease circuit implementation of BPSDMs.

REFERENCES


