Stability analysis for a class of switched large-scale time-delay systems via time-switched method

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Abstract: One new criterion of delay-independent stability for the switched time-delay large-scale system is deduced by employing a time-switched method and the comparison theorem. The total activation time ratio of the switching law can be determined to guarantee that the switched time-delay system is exponentially stable with stability margin \( \lambda \). One example is exploited to illustrate the proposed schemes.

1 Introduction

Switched systems are an important class of hybrid systems. Such systems can be described by a family of continuous-time subsystems (or discrete-time subsystems) and a rule that orchestrates the switching between them; for example, a given process exhibits a switching behaviour caused by abrupt changes of the environment. In the control process of practical systems, over and above two subsystems are required to be switched, such as power systems, chemical procedure control systems, navigation systems, automobile speed change systems and so on. In the study of switched systems, most works have been centralised on the problem of stability. In the last two decades, there has been increasing interest in stability analysis and stabilisation of such switched systems [1–3]. Two important methods are used to construct the switching law for the stability analysis of the switched systems. One is the state variable method and the other is the time-switched method. The former is that, if all subsystems mainly have a common Lyapunov function, they have a choice of many switching strategies to make the whole system stable. However, many switched systems do not have a common Lyapunov function and have received a lot of attention in the quest for a restricted class of switching laws under which switched systems are stable. The latter is the main concept of dwell-time. Recently, with regard to the stability analysis of switched systems, an analysis method of dwell-time has proved particularly effective and is described in various studies [4–9]. Moreover, these research results can provided a lot of references for the academia and industry.

When all subsystem matrices are Hurwitz stable, then the entire switched system is exponentially stable for any switching signal if the time between consecutive switching (dwell-time) is sufficiently large [4]. Switching between stable linear systems results in a stable system provided that switching is slow-on-the-average [5]. But in many applications, unstable subsystems of switched systems cannot be avoided. If the average dwell-time chosen is sufficiently large and the total activation time of unstable subsystems is relatively small compared with that of Hurwitz stable subsystems, then exponential stability of a desired degree is guaranteed in the work of Zhai et al. [6, 7]. Lee et al. [8] have proposed stability properties, using the concepts of minimum/maximum holding time and redundancy of each engaged subsystem instead of the multiple Lyapunov functions. A different kind of ‘time-varying’ or ‘adjustable’ dwell-time to deal with the presence of disturbances for switched nonlinear systems is given by Persis et al. [9].

Furthermore, the time-delay phenomenon also cannot be avoided in practical systems; for instance, chemical processes, long distance transmission line, hybrid procedure, electron networks and so on. The problem of time-delay is that of instability and poor performance of practical systems [10, 11]. Therefore the stability analysis of switched time-delay systems is worth being researched. Until recently, there has been little in the literature [12, 13] concerning this problem. Besides, a large-scale system is often considered as a set of interconnected subsystems. The advantage of this aspect in stability analysis is to reduce complexity [14]. Recently, many approaches have been used to investigate the stability and stabilisation of large-scale time-delay systems [15, 16]. Therefore the stability analysis of switched large-scale time-delay systems is worth researching.

In this paper, in view of the time-switched method and the comparison theorem, sufficient stability conditions with delay-independence will be derived for the switched large-scale time-delay systems. The total activation time ratio under the switching laws should not be less than a specified constant. Finally, simulation examples are given to demonstrate our result. The following notations will be used throughout the paper: \( \lambda(A) \) stands for the eigenvalues of matrix \( A \), \( \|A\| \) denotes the norm of matrix \( A \), that is, \( \|A\| = \max\{\lambda(A^T A)^{1/2}\} \) and \( \mu(A) \) means the matrix measure of matrix \( A \), that is, \( \mu(A) = \max\{\lambda(A + A^T)/2\} \).

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2 System description and problem statement

Consider the switched large-scale time-delay systems

$$\dot{x}_i(t) = A_{ij}x_i(t) + \sum_{j=1}^{N} A_{ij}x_j(t) + \sum_{j=1}^{N} B_{ij}x_j(t - \tau(t)) + N \sum_{j=1}^{N} B_{ij}x_j(t - \tau(t))$$

where $x \in \mathbb{R}^n$, $A$ and $B$ are matrices in proper dimensions and $\tau$ is the delay duration.

Lemma 1 [17]: The stability of system (13) implies the stability for the following systems

$$\dot{y}(t) = (A + zB)y(t), \quad \forall |z| = 1$$

and vice versa. Where $z = \exp(j\theta), \theta \in [0, 2\pi], j = \sqrt{-1}$ and $z \in \mathbb{C}$.

Lemma 2 [18]: For matrix $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times n}$, the following relation holds

$$\|\exp(A + zB)\| \leq \exp(\|A\| + |B|t). \quad \forall |z| = 1$$

In the light of Lemma 1, it is obvious that the stability of the switched large-scale time-delay system (1) is equivalent to that of the switched large-scale system

$$\dot{w}_i(t) = A_{ij}w_i(t) + \sum_{j=1}^{N} A_{ij}w_j(t) \quad \forall |z| = 1$$

where $A_{ij} = A_{ij} + zB_{ij}$ and $A_{ij}w_j(t) + zB_{ij}$.

Without loss of generality, we assume that the switched large-scale time-delay system (1) at least has one individual switched system whose $\|A_{ij}\| + \|B_{ij}\|$ values, $i = 1, 2, \ldots, N$, are all less than zero, and

$$\lambda_i^* = \min_{1 \leq j \leq N} (\|A_{ij}\| + \|B_{ij}\|)$$

when $\|A_{ij}\| + \|B_{ij}\| < 0$ (6a)

$$\lambda_i^* = \max_{j \neq i} (\|A_{ij}\| + \|B_{ij}\|)$$

when $\|A_{ij}\| + \|B_{ij}\| \geq 0$ (6b)

we assume $T_i(t)$ (or $T_i^+(t)$) is the total activation time of the $i$th subsystem whose $\|A_{ij}\| + \|B_{ij}\|$ values are no less than zero (total activation time of the $i$th subsystem whose $\|A_{ij}\| + \|B_{ij}\|$ values are less than zero). The switching law of the $i$th subsystem can be defined as follows

$$\inf_{t \geq 0} \left[ \frac{T_i(t)}{T_i^+(t)} \right] \geq \frac{\lambda_i^*}{\lambda_i^* - \lambda_{ij}^*}$$

where $\lambda_i \in (0, \lambda_i^*), \lambda_{ij} \in (\lambda_i, \lambda_i^*), 1 \leq i \leq N$.

Furthermore, we assume $T_{i1}$ (or $T_i^+$) is the total activation time of individual switched systems whose $\|A_{ij}\| + \|B_{ij}\|$ values are all less than zero for $i = 1, 2, \ldots, N$ (the total activation time of individual systems at least has one subsystem whose $\|A_{ij}\| + \|B_{ij}\|$ values are less than zero). The total activation time ratio between $T_{i1}$ and $T_i^+$ can be called a switching law of the switched time-delay large-scale system (1).

Therefore this paper is concerned with the following problem: to find the total activation time ratio between $T_{i1}$ and $T_i^+$, which guarantees that the switched large-scale time-delay system (1) is globally exponentially stable with stability margin $\lambda_i$.

Furthermore, we define an auxiliary matrix as following

$$y(t) = Cy(t)$$

where

$$y(t) = [y_1(t) \ y_2(t) \ \cdots \ y_N(t)]^T \in \mathbb{R}^N, \quad y_i(t) \in \mathbb{R}$$

and $C = [c_{ij}] \in \mathbb{R}^{N \times N}, c_{ij} = -\lambda_i, \ i = j; c_{ij} = 0, i \neq j$ for $i, j = 1, 2, \ldots, N$, where $\bar{A}_i^* = \max_{l=1, \ldots, r} (\|A_{il}\| + \|B_{il}\|)$.

A sufficient condition for stability of the switched large-scale time-delay systems (1) is established in the following theorem by using the time-switched method.

3 Main result

Theorem 1: Suppose the switched large-scale time-delay system (1) has at least one individual switched system whose $\|A_{ij}\| + \|B_{ij}\|$ values, $i = 1, 2, \ldots, N$, are all less than zero. The switched large-scale time-delay system (1) is globally exponentially stable with stability margin $\lambda_i$, if the system (8) is stable and the system (1) satisfies the following switching law

$$\inf_{t \geq 0} \left[ \frac{T_{i1}(t)}{T_i^+(t)} \right] \geq \max_{1 \leq i \leq N} \left[ \frac{T_{i1}(t)}{T_i^+(t)} \right]$$

where $\lambda_i = \min_{1 \leq i \leq N} (\lambda_i)$.

Proof: The trajectory response of system (5) can be written as follows

$$w_i(t) = e^{A_{ij}t}e^{\int_{t_0}^{t} d\tau} e^{A_{ij}e^{\int_{t_0}^{\tau} d\tau - \tau} \cdots e^{A_{ij}e^{\int_{t_0}^{\tau} d\tau - \tau}}}w_i(t_0)$$

$$+ \int_{t_0}^{t} e^{A_{ij}e^{\int_{t_0}^{\tau} d\tau} e^{\int_{t_0}^{\tau} d\tau} \cdots e^{A_{ij}e^{\int_{t_0}^{\tau} d\tau - \tau}}} \sum_{j=1}^{N} A_{ij}w_j(s) ds$$

$$+ \int_{t_0}^{t} e^{A_{ij}e^{\int_{t_0}^{\tau} d\tau} e^{\int_{t_0}^{\tau} d\tau} \cdots e^{A_{ij}e^{\int_{t_0}^{\tau} d\tau - \tau}}} \sum_{j=1}^{N} A_{ij}w_j(s) ds$$

$$+ \int_{t_0}^{t} e^{A_{ij}e^{\int_{t_0}^{\tau} d\tau} e^{\int_{t_0}^{\tau} d\tau} \cdots e^{A_{ij}e^{\int_{t_0}^{\tau} d\tau - \tau}}} \sum_{j=1}^{N} A_{ij}w_j(s) ds$$

where $p_j \in \{1, 2, \ldots, N\}$. 
Let
\[ v_j(t) = e^{\lambda_j t}(t-t_0) e^{\lambda_j(t_0-t_{j-1})} \cdots e^{\lambda_j t_{j-1}} y_j(t_0) \] (11)

Performing the norm on both sides of (11) and in view of Lemma 2, we can obtain
\[ \| v_j(t) \| \leq e^{|\lambda_j t_0|} e^{\lambda_j(t-t_0)} e^{\lambda_j(t_0-t_{j-1})} \cdots e^{\lambda_j t_{j-1}} \| y_j(t_0) \| \]
\[ \cdots e^{\lambda_j t_{j-1}} \| y_j(t_0) \| \]
\[ \leq e^{\lambda_j t_0} e^{\lambda_j(t-t_0)} e^{\lambda_j(t_0-t_{j-1})} \cdots e^{\lambda_j t_{j-1}} \| y_j(t_0) \| \]
\[ \leq e^{\lambda_j (t-t_0) - \lambda_i (t_i - t_{i-1})} \| v_j(t_0) \| \] (12)

Furthermore, the switching law (7) of the ith subsystem means
\[ \lambda_i^+ t_i^+(t) - \lambda_i^- t_i^-(t) \leq -\lambda_i^+ (t_i^+(t) + t_i^-(t)) \]
\[ = -\lambda_i^-(t_i - t_0) \] (13)

Hence, the following inequality can be obtained
\[ \| v_i(t) \| \leq e^{-\lambda_i (t-t_0)} \| v_i(t_0) \| \]
and
\[ \| w_j(t) \| \leq e^{-\lambda_i (t-t_0)} \| w_j(t_0) \| \]
\[ + \int_{t_0}^t e^{-\lambda_i (t-s)} \sum_{j \neq i} a_{ji}^+ \| w_j(s) \| ds \]
\[ + \int_{t_0}^t e^{-\lambda_i (t-s)} \sum_{j \neq i} a_{ji}^- \| w_j(s) \| ds \]
\[ + \cdots + \int_{t_0}^t e^{-\lambda_i (t-s)} \sum_{j \neq i} a_{ji}^+ \| w_j(s) \| ds \]
\[ = e^{-\lambda_i (t-t_0)} \| w_i(t_0) \| \]
\[ + \int_{t_0}^t e^{-\lambda_i (t-s)} \sum_{j \neq i} a_{ji}^- \| w_j(s) \| ds \] (14)

Let
\[ y_i(t) = e^{-\lambda_i (t-t_0)} \| y_i(t_0) \| + \int_{t_0}^t e^{-\lambda_i (t-s)} \sum_{j \neq i} a_{ji}^- \| y_j(s) \| ds \] (15)

Then, (15) is the solution of system (16)
\[ \dot{y}_i(t) = -\lambda_i y_i(t) + \sum_{j \neq i} a_{ji}^- y_j(t) \] (16)

Hence, the exponential stability of \( y_i(t) \) implies that of \( w_i(t) \) by the comparison theorem [19].

Furthermore, system (16) can also be expressed as system (8). It is clear that if system (8) is stable and the switching law is as (9), then the switched large-scale time-delay system (1) is also globally exponentially stable with stability margin \( \lambda \).
Switched system 3 (l = 3)

\[
\dot{x}_1(t) = [-8 \ 0 \ 0] x_1(t) + [-0.2 \ 0.3 \ 0.2 \ 0.5 \ 0.1 \ 0.3] x_2(t) + [0.1 \ 0.4 \ 0.1 \ 0.4 \ 0.7 \ -0.1] x_3(t) + [0.1 \ -0.1 \ 0.2 \ 0.4 \ 0.1 \ 0.2 \ 0.5 \ 1 \ 1] x_1(t - \tau)
\]

\[
\dot{x}_2(t) = [1 \ 0.4 \ -1 \ -0.1 \ 0.1 \ 0.4 \ 0.6 \ 1 \ 0.7 \ 0.1 \ 0.8 \ -0.1 \ 0.2 \ 0.1 \ 1] x_3(t - \tau) + [0.1 \ -0.2 \ 0.1 \ -0.1 \ 0.5 \ 0.8 \ -0.1] x_1(t - \tau) + [0.2 \ 0.4 \ 0.1 \ 0.5 \ -1 \ 0.1] x_2(t - \tau) + [-0.1 \ -0.1 \ 0.4 \ 0.1 \ 0.1 \ -2 \ -1 \ 0 \ 0.5] x_3(t - \tau)
\]

\[
\dot{x}_3(t) = [-0.1 \ -0.1 \ 0.5 \ 1 \ 0.2 \ 0.3 \ 0.1 \ 0.2 \ 0.5 \ 0.1 \ -0.5 \ 0.2 \ 0.3 \ -0.1 \ 0.8 \ -0.1 \ 1] x_1(t) + [0.4 \ 0.1 \ 0.1 \ 0.5 \ -1 \ 0.1] x_2(t) + [0.5 \ 0.1 \ -1 \ -2 \ 0.1 \ 0.1] x_2(t - \tau) + [0.4 \ 0.1 \ 0.1 \ 0.5 \ -1 \ 0.1] x_1(t - \tau)
\]

(17c)

According to the normal test of stability for the large-scale system, the switched system 1 is stable, whereas the switched systems 2 and 3 are both unstable. We can easily calculate \( r_1 = 7.6382, \quad r_2^1 = 3.1088, \quad r_2 = 8.3814, \quad r_2^2 = 3.3419, \quad r_3 = 8.1258 \) and \( r_3^1 = 2.9037. \) Hence, the stable matrix

\[
c = \begin{bmatrix} -5 & 2.8021 & 2.6298 \\ 2.3730 & -6 & 2.6822 \\ 3.5487 & 3.3522 & -6.5 \end{bmatrix}
\]

can be obtained by choosing \( r_1 = 5, \quad r_2 = 6 \) and \( r_3 = 6.5. \) Furthermore, the switching law of the \( i \)th subsystem, \( i = 1, 2, 3, \) can be calculated as \( T_i/r_i > 3.0736, \quad T_2^1/T_2 > 3.9229, \quad T_3/T_3^1 > 5.784. \) Therefore the total activation time ratio for the switching law is

\[
\frac{T^*(t)}{T^0(t)} > 5.784
\]

(18)

Finally, the switched large-scale time-delay system (17) is globally exponentially stable with stability margin \( \lambda \in (0, 5) \) under the switching law (18). In order to satisfy the switching law (18), we choose the total activation time ratio as 6 : 1. The activation time of individual system 1 is 1.2 s and the total activation time of individual systems 2 and 3 is 0.1 s. The trajectory of the time-delay switched system is shown in Fig. 1 with initial state \( x_i(t) = [1 \ 2]^T \) for \( i = 1, 2, 3 \) and time-delay \( \tau \) is 0.1 s.

5 Conclusion

According to the time-switched method and the comparison theorem, the sufficient conditions of the switched laws are presented and the total activation time ratio under the switching laws is required to be no less than a specified constant, such that the switched large-scale time-delay system is delay-independent exponentially stable with stability margin \( \lambda. \) The problems of designing the switched laws with time-dependent for the switched large-scale time-delay system remain open and are interesting topics for future research.

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7 References


